Solutions to CSC 165H Afternoon Midterm 2003

Question 1. [3 MARKS]

Whether or not the Riemann Hypothesis is true is currently unknown. Suppose a researcher proves that:

(*) The Riemann Hypothesis implies S.

Part (a) [2 MARKS] What does (*) tell us if it is also proven that:

1. S is true.

   nothing

2. S is false.

   The Riemann Hypothesis is false.

Part (b) [1 MARK] What could we find out about the Riemann Hypothesis that would make (*) useful?

The Riemann Hypothesis is true.

Question 2. [5 MARKS]

Consider the following, where p, q and r are sentences.

(S) p and q are equivalent only if r.

Part (a) [1 MARK] Express (S) as a sentence in our precise language.

Another way to express this in English is "If p and q are equivalent then r". From this, it is easy to get the following sentence in our precise language:

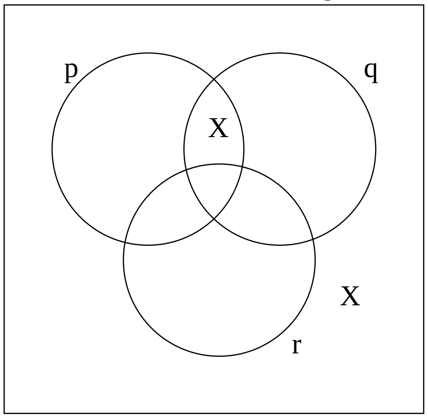
$$(p \iff q) \implies r.$$

A common error was confusing "if" and "only if" : $r \implies (p \iff q)$.

Part (b) [2 MARKS] Express (S) as a sentence in our precise language without using $\Rightarrow$ or $\Leftrightarrow$ (you do not have to make it simple).

Two equivalent correct answers are:
$(p \wedge \sim q) \vee (\sim p \wedge q) \vee r$
and $\sim [(p \wedge q) \vee (\sim p \wedge \sim q)] \vee r$.

Part (c) [2 MARKS] Draw a Venn diagram with sets for p, q and r. Make sure the sets overlap to divide the diagram into eight regions. Shade in the regions corresponding to where your sentence from (a) is true, and put an "X" in the regions where it is not true.



Note: even if your answer to part (a) was wrong, you got full marks for part (b) if your Venn diagram correctly described your answer.

Question 3. [3 MARKS]

Express the sentence

Program F is faulty if some input causes it to crash.

in our precise language, using the domains:

P = the set programs.
I = the set of inputs.

Define any properties you need.

Let $f(p)$ = "program p is faulty"
and $c(x, p)$ = "input x causes program p to crash".
$(\exists x \in I, c(x, F)) \implies f(F)$

Question 4. [6 MARKS]

Consider the results of a contest.

Let C = the set of contestants.
Let $beat(c1, c2)$ = "contestant $c1$ scored more than contestant $c2$".

Assume every contestant got a different score.

Part (a) [3 MARKS] Using C and $beat$, give a sentence in our precise language that is equivalent to:

"Contestant c won the contest"

$\forall x \in C, ((\sim (x = c)) \implies beat(c, x))$

Two incorrect answers are:
$\forall c \in C, \forall x \in C, (\sim (x = c)) \implies beat(c, x))$ and

$\exists c \in C, \forall x \in C, (\sim (x = c)) \implies beat(c, x))$.
These are wrong because c is an unquantified variable in the sentence *"Contestant c won the contest"*.

Two other incorrect solutions are:
$\forall x \in C, beat(c, x)$
and $\forall x \in C, ((\sim (x = c)) \wedge beat(c, x))$.

Part (b) [3 MARKS] Using C and *beat*, give a sentence in our precise language that is equivalent to:

"Contestant c came in second-place"

$\exists w \in C, beat(w, c) \wedge \forall x \in C, ((x \neq c \wedge x \neq w) => beat(w, x))$
Similarly, in this solution, c must be unquantified.

Question 5. [8 MARKS]

Consider the following sentence about a sequence of natural numbers $a_1, a_2, a_3, \ldots$:

$$(*)\ \exists i \in N, \forall j \in N, a_j = a_i \implies a_{j+1} \neq a_i$$

Part (a) [4 MARKS] Give the outline of a proof structure for the sentence.

Let $i = \cdots$
Then $i \in N$.
  Let j be an arbitrary element of N.
    Suppose $a_j = a_i$.
    $\vdots$
    Thus $a_{j+1} \neq a_i$.
    Hence $a_j = a_i \implies a_{j+1} \neq a_i$.
  Since j is an arbitrary element of N,
  $\forall j \in N, a_j = a_i \implies a_{j+1} \neq a_i$.
Hence $\exists i \in N, \forall j \in N, a_j = a_i \implies a_{j+1} \neq a_i$.

Part (b) [4 MARKS] For each of the following sequences, state whether the sentence (*) is true or false:
The sentence means: There is some number that occurs in the sequence, but never occurs twice in a row.

1. $1, 2, 2, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, \ldots$
   true
2. $1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, 1, \ldots$
   true
3. $1, 2, 1, 1, 2, 2, 1, 1, 1, 2, 2, 2, 1, 1, 1, 1, 2, 2, 2, 2, 1, \ldots$
   false
4. $1, 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, \ldots$
   false