Solutions to CSC 165H Morning Midterm 2003

Question 1. [3 MARKS]

Whether the statement "P = NP" is true or not is unknown. Suppose a researcher proves that:

(*) If P = NP then S.

Part (a) [2 MARKS] What does (*) tell us if one day it is proven that:

1. P = NP

   S is true.

2. P $\neq$ NP

   nothing

Part (b) [1 MARK] What could we find out one day about S that would make (*) useful?

S is false.

Question 2. [5 MARKS]

Consider the following, where p, q and r are sentences.

(S) If p then q, otherwise r.

Part (a) [3 MARKS] Express (S) as a sentence in our precise language.

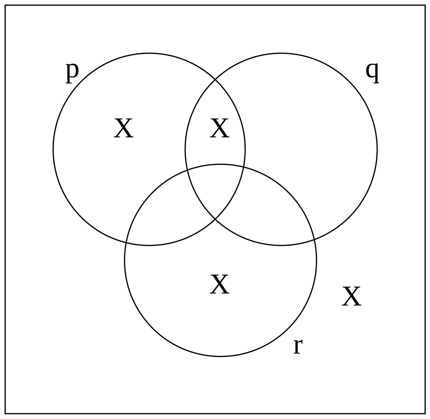
Another way to express this in English is "If p then q and if not p then r". From this, it is easy to get the following sentence in our precise language:

$$(p \implies q) \wedge (\sim p \implies r).$$

An equivalent solution is $(p \wedge q) \vee (\sim p \wedge r)$.

Two common incorrect solutions were:
$(p \implies q) \vee r$ and
$p \implies q \vee r$

Part (b) [2 MARKS] Draw a Venn diagram with sets for p, q and r. Make sure the sets overlap to divide the diagram into eight regions. Shade in the regions corresponding to where your sentence from (a) is true, and put an "X" in the regions where it is not true.



Note: even if your answer to part (a) was wrong, you got full marks for part (b) if your Venn diagram correctly described your answer.

Question 3. [3 MARKS]

Express the sentence

No process that wants the resource can take a step.

in our precise language, using the domain

P = the set of all processes.

Define any properties you need.

Let $w(p)$ = "process p wants the resource".
Let $s(p)$ = "process p can take a step".
$\sim \exists p \in P, w(p) \wedge s(p)$.

Another correct sentence is $\forall p \in P, w(p) \implies \sim s(p)$.

A common incorrect solution was: $\forall p \in P, \sim w(p) \implies s(p)$.

Question 4. [6 MARKS]

Consider a company with a president, vice-president, and various other employees, that has a chain of command:

President
Vice-president
Senior project manager
...

Each employee obeys themself and everyone higher up in the chain (but no one else). For example: the senior project manager obeys herself, the vice-president, the president, and no one else.

Let E = the set of employees in this company.
Let $obeys(e1, e2)$ = "employee $e1$ obeys employee $e2$".

Part (a) [3 MARKS] Using E and *obeys* (but not the constants "President", "Vice-president", or "Senior project manager"), give a sentence in our precise language that is equivalent to:

> *"Employee e is the president"*

$\forall x \in E, obeys(x, e)$

Two incorrect answers are:
$\forall e \in E, \forall x \in E, obeys(x, e)$ and
$\exists e \in E, \forall x \in E, obeys(x, e)$.
These are wrong because e is an unquantified variable in the sentence *"e is the president"*.

Part (b) [3 MARKS] Using E and *obeys* (but not the constants "President", "Vice-president", or "Senior project manager"), give a sentence in our precise language that is equivalent to:

> *"Employee e is the vice-president"*

$\exists p \in E, (p \neq e \wedge obeys(e, p) \wedge \forall x \in E, (x \neq p \implies obeys(x, e)))$

Similarly, in this solution, e must be unquantified.

Question 5. [8 MARKS]

Consider the following sentence about a sequence of natural numbers $a_1, a_2, a_3, \ldots$:

$$\forall i \in N, \exists j \in N, a_i = a_j \wedge a_j = a_{j+1}$$

Part (a) [4 MARKS] Give the outline of a proof structure for the sentence.

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Let $i$ be an arbitrary natural number.
    Let $j = \cdots$.
    Then $j \in N$.
    $\vdots$
    $a_i = a_j$
    $\vdots$
    $a_j = a_{j+1}$
    Therefore $\exists j \in N, a_i = a_j \wedge a_j = a_{j+1}$.
Since $i$ is an arbitrary element of $N$,
$\forall i \in N, \exists j \in N, a_i = a_j \wedge a_j = a_{j+1}$.
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Part (b) [4 MARKS] For each of the following sequences, indicate whether the sentence is true or false:

1. $1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \ldots$
   false
2. $4, 4, 3, 3, 2, 2, 1, 1, 4, 3, 2, 1, 1, 1, 1, 1, \ldots$ (all the rest are ones)
   true
3. $1, 2, 1, 1, 2, 2, 1, 1, 1, 2, 2, 2, 1, 1, 1, 1, 2, 2, 2, 2, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 1, \ldots$
   true
4. $1, 1, 1, 2, 3, 3, 2, 1, 1, 1, 2, 3, 3, 2, 1, 1, 1, 2, 3, 3, 2, \ldots$
   false