Problem Set 3

CSC 463

Due by March 13, 2020, 2pm

Each problem set counts for 10% of your mark. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly. You may cite results discussed in lecture or the course textbook. You are encouraged to write precisely and concisely; it should be possible to write your solutions to the problem set within a page per question. Submit your work online by uploading a pdf file or image of your solutions onto Crowdmark. In all problems, you can assume that you are given valid encodings as input to the problem.

1. A language L is in **coNP** if the complement $\bar{L} \in$ **NP.** It is generally believed that **NP** $\neq$ **coNP**, which implies that **P** $\neq$ **NP**.

   A Boolean formula ϕ is a **tautology** if ϕ evaluates to true on every possible truth assignment. Let **TAUT** $= \{\langle \phi \rangle | \phi$ is a tautology$\}$ be the set of Boolean tautologies. Show that **TAUT** is **coNP**-Complete (i.e. **TAUT** $\in$ **coNP** and there is reduction $\mathbf{A} \leq_{\mathbf{p}}$ **TAUT** for all $A \in$ **coNP**).

2. A **partial 3-colouring** of a graph G is a map $g : V' \mapsto \{1, 2, 3\}$ such that $g(u) \neq g(v)$ for vertices $u, v \in V'$ in some subset $V' \subset V$ provided (u, v) is an edge in G. We say that g **extends** to a full 3-colouring of G if there is a 3-colouring f covering all vertices of G such that $f(u) = g(u)$ for all $u \in V'$. Let **PARTIAL-3COL** be the problem of determining given $\langle G, g \rangle$ where g is a partial 3-colouring of G, whether or not g extends to a full 3-colouring of G.

   Show that there is a reduction **PARTIAL-3COL** $\leq_p$ **3COL**. Therefore, there is an efficient decision-to-search reduction for **3COL.**

3. A Boolean formula ϕ is **nice** if it is in conjunctive normal form, and each clause consists entirely of unnegated or negated variables. Let **NICE-SAT** be the problem of determining if a nice Boolean formula is satisfiable or not. Prove that **NICE-SAT** is **NP**-Complete.

4. Given a graph G, we say that a subset of its vertices $S \subset V$ is **triangle-free** if for every size three subset $\{u, v, w\} \subset S$, at least one of the edges uv, vw, uw is not in the graph G. Let **TF** be the problem of determining if a graph G has a triangle-free subset of size at least k. Show that **TF** is **NP**-Complete by reducing from independent set.

5. (Optional/ungraded: for extra knowledge and extra practice) Recall a weighted graph is a graph with a function $w : E \mapsto \mathbb{Z}^+$ assigning weights to its edges. Given a graph G, a **cut** is a division of the vertices into disjoint sets (S, T) with $S \cup T = V$. An edge crosses the cut if one of its endpoints is in S and the other endpoint is in T. The weight of a cut in a weighted graph is the sum $\sum_{e \in E} w(e)$ over all edges e crossing the cut. Given a weighted graph, show that the problem of determining if it has a cut of weight at least k is **NP**-Complete. This problem is related to the Ising model of magnetism studied in physics.