The "tree-dependent" components of natural scenes are edge filters

Daniel Zoran and Yair Weiss

CifAR NCAP Summer School – August 2010

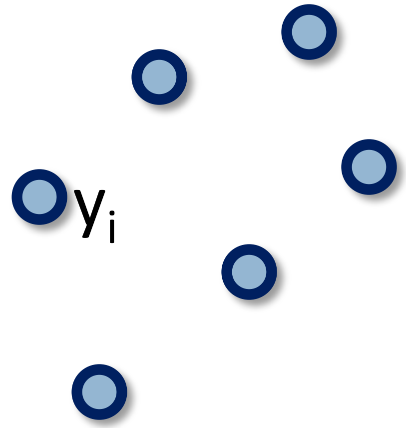
Introduction

ICA

- Maximize independence of filter outputs
- Equivalent to ML with the model:

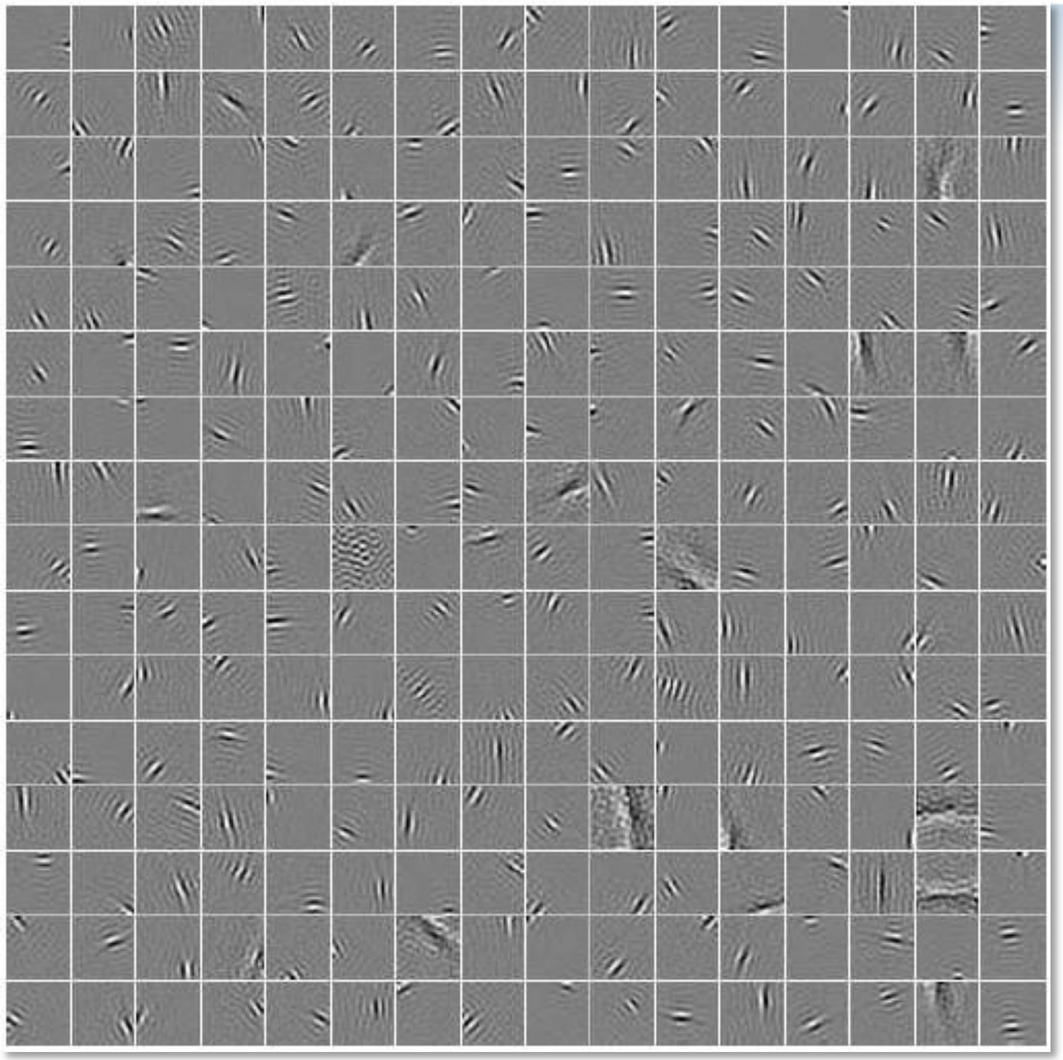
$$P(\mathbf{x};\mathbf{W}) = \prod_i p(y_i)$$

$$\mathbf{y} = \mathbf{W}\,\mathbf{x}$$



ICA on Natural Images

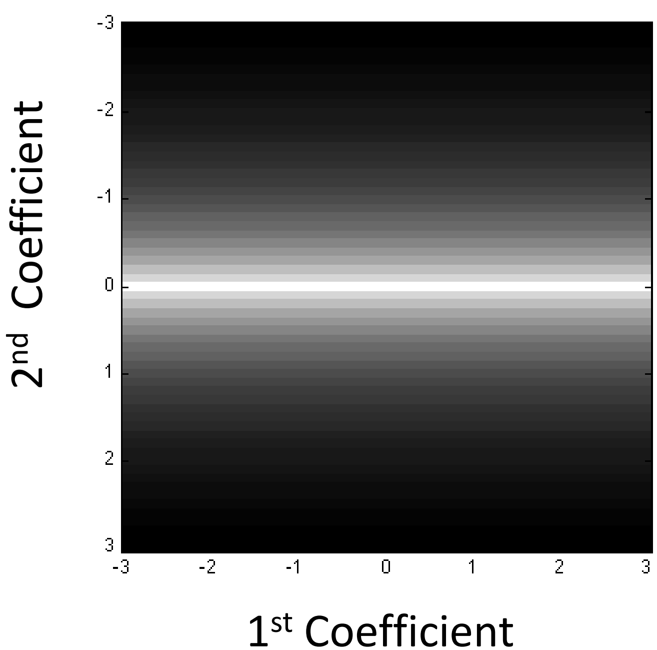
Bell & Sejnowski
Olshausen & Field



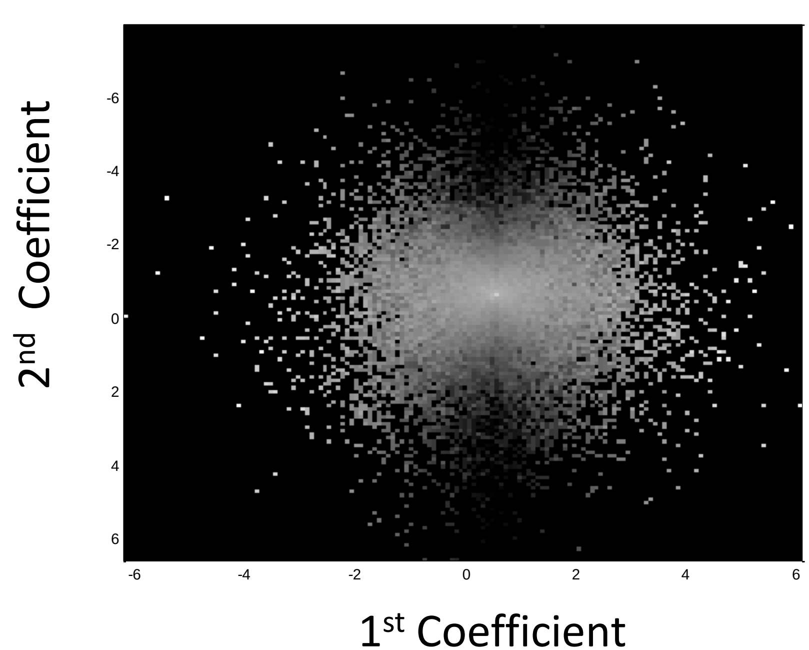
Problems with ICA

- Components are not *really* independent

Independent Coefficients



Dependent Coefficients - **Bowtie**

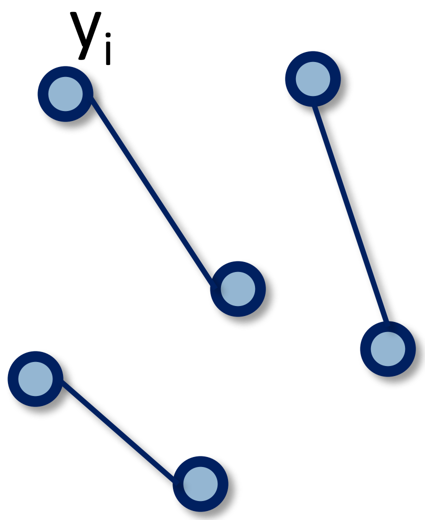


Independent Subspace Analysis

- Independent *subspaces* of the data

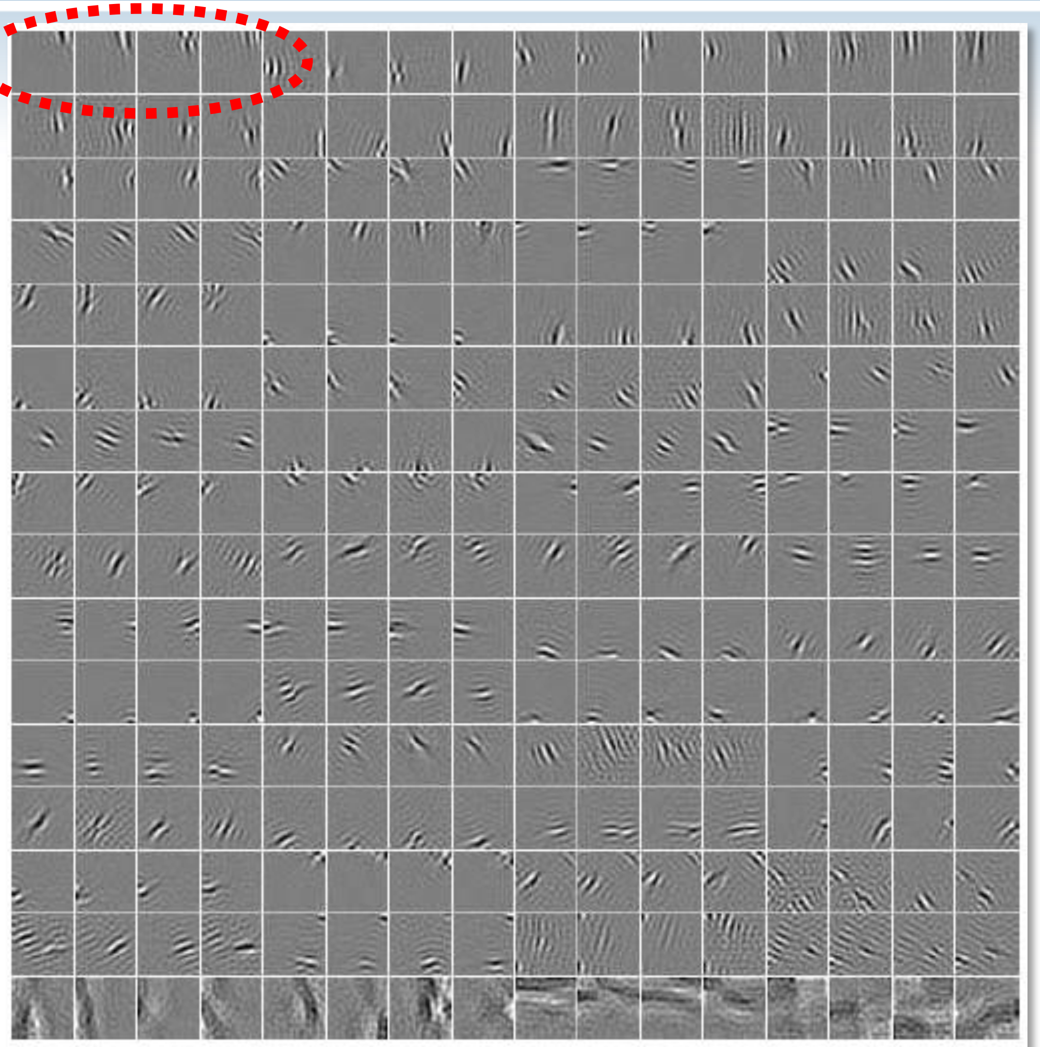
$$P(\mathbf{x}) = \prod_{k} p\left(\sum_{i \in K} y_i^2\right)$$

$$\mathbf{y} = \mathbf{W}\mathbf{x}$$



ISA on Natural Images

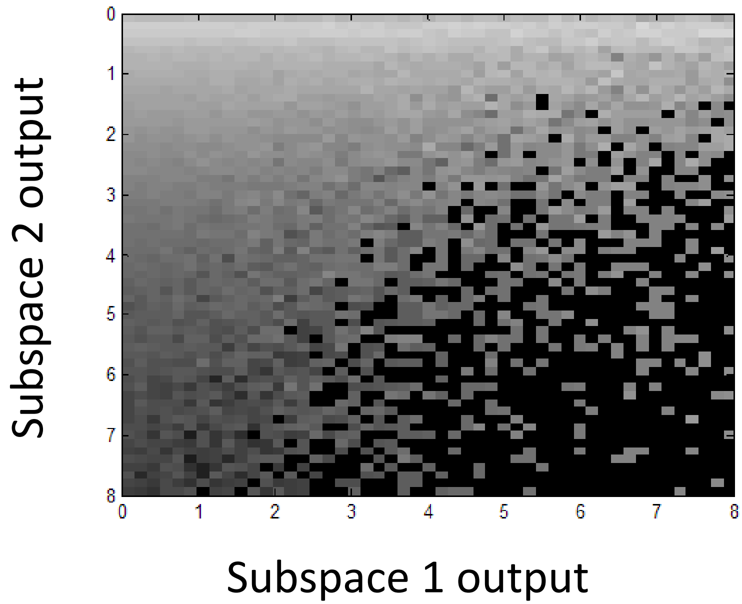
“Explains” complex cells



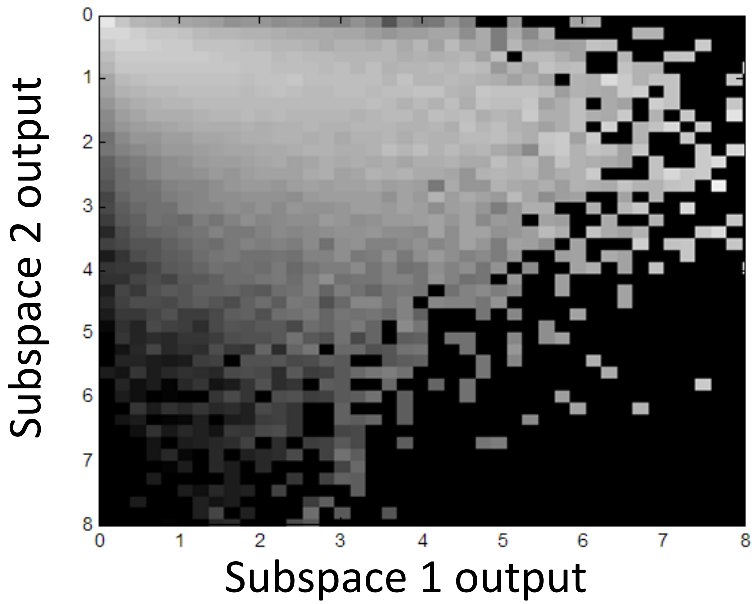
Dependencies Between Subspace Energies

Conditional histogram of subspace outputs

Synthetic independent data

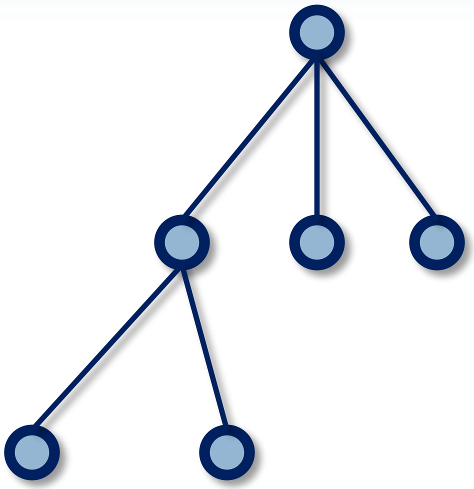


ISA from natural images



Our model

- Our model *assumes* tree dependency



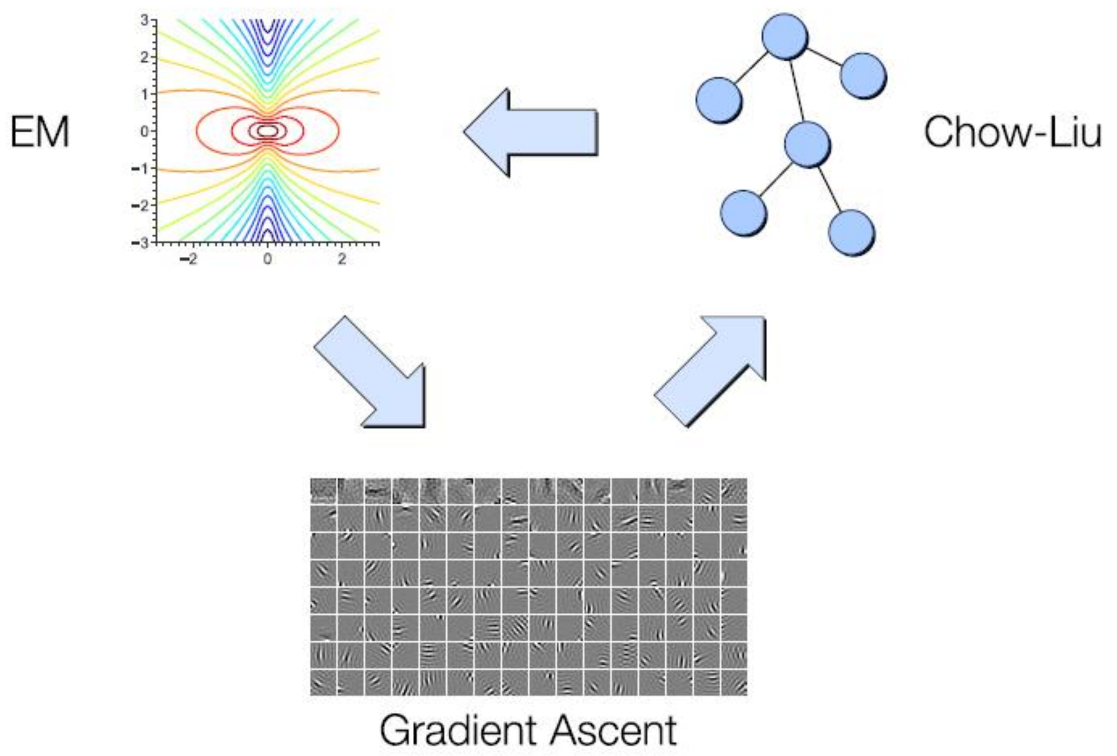
$$P(\mathbf{x};\mathbf{W}) = p(y_{root}) \prod_{i \neq root} p(y_i \mid y_{pa(i)})$$

$$\mathbf{y} = \mathbf{W}\mathbf{x}$$

Model and learning

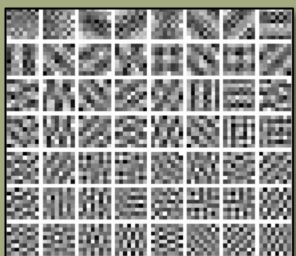
Model and learning

$$p(\mathbf{z};\mathbf{W},\beta,T) = p(y_1)\prod_{i=2}^{N} p(y_i \mid y_{pa(i)};\beta) \qquad \mathbf{y} = \mathbf{W}\mathbf{z}$$

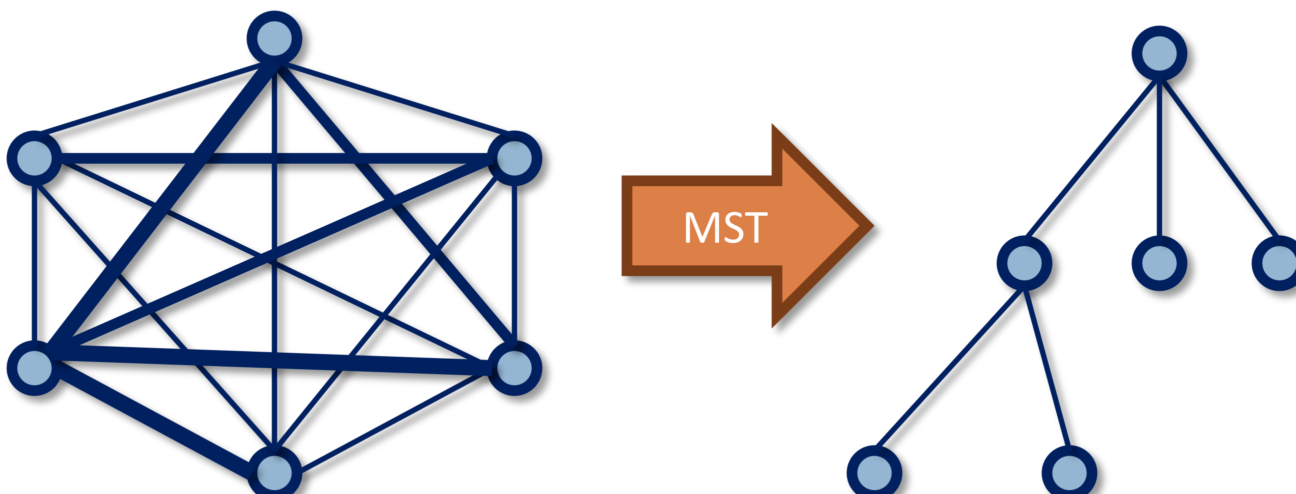


Learning Tree Structure

Constant

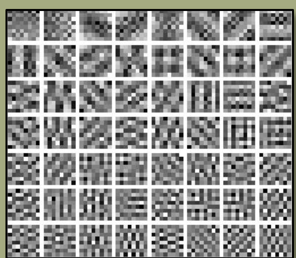


- A current estimate for **W** is given
- Chow-Liu method

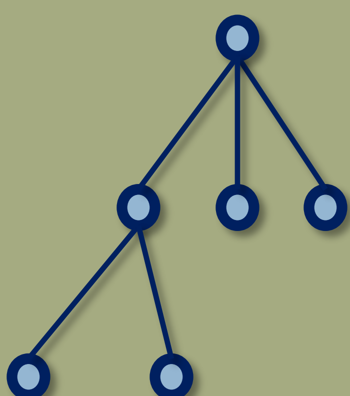


Joint Pairwise Density Function

Constant



Constant



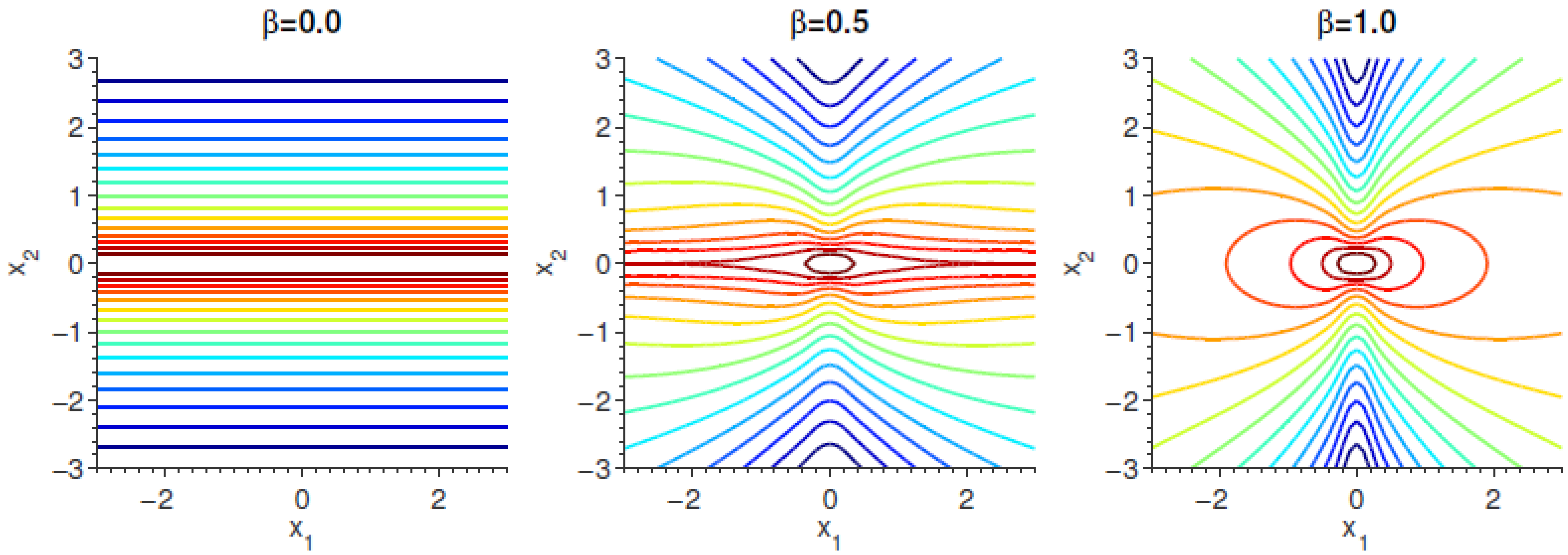
- Mixture model – allows both dependence and independence

$$p(y_1, y_2; \theta) = \beta\, p_1(y_1, y_2; \theta) + (1 - \beta)\, p_2(y_1; \theta)\, p_2(y_2; \theta)$$

- Mixing variable learned from data using EM

Joint Pairwise Density Function (cont.)

- GMM for densities
- Captures highly kurtotic shape of coefficients



Learning the filter matrix

- We assume $\mathbf{W} = \mathbf{R}\mathbf{V}$ where $\mathbf{V}$ is a whitening transform and $\mathbf{R}$ is a rotation matrix
- We use $\mathbf{V}$ to first whiten the patches so that:

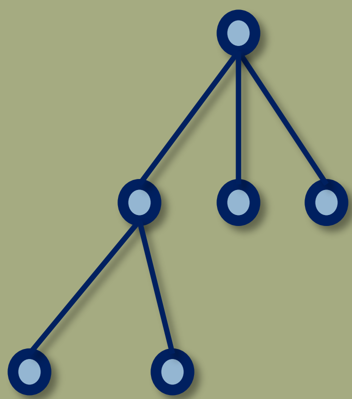
$$\mathbf{z} = \mathbf{V}\mathbf{x} \qquad < \mathbf{z}\mathbf{z}^T >= \mathbf{I}$$

$$\mathbf{y} = \mathbf{R}\mathbf{z}$$

- Now we need to learn the matrix $\mathbf{R}$

Learning the filter matrix

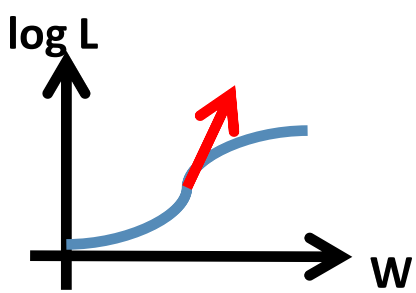
Constant

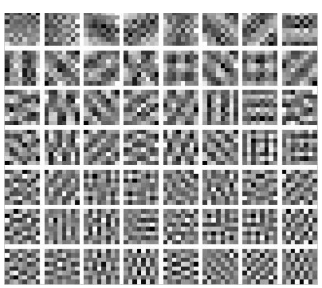


- A current estimate for the tree structure is given
- Gradient Ascent on log likelihood:

$$\mathbf{R}_r^{t+1} = \mathbf{R}_r^{t} + \eta \frac{\partial \log p(\mathbf{y})}{\partial y_r} \mathbf{z}^T$$

- Impose orthogonality: $\mathbf{R} = (\mathbf{R}\mathbf{R}^T)^{-0.5}\mathbf{R}$





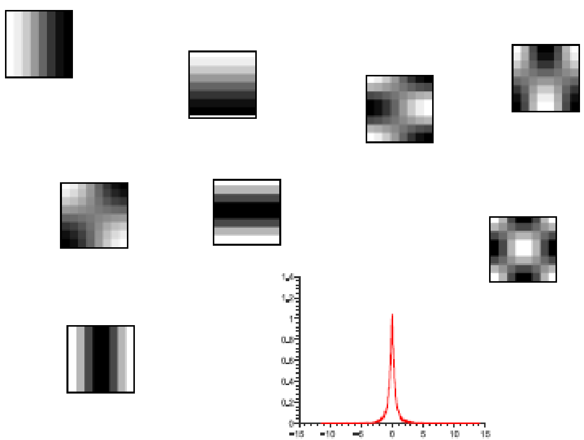
Learning details

- Learning in mini-batches
- Iterate:
  - Perform Gradient Ascent
  - Every 500 mini-batches, relearn tree structure and parameters
- Alternative method for Tree Component Analysis - Bach et al. [2004]

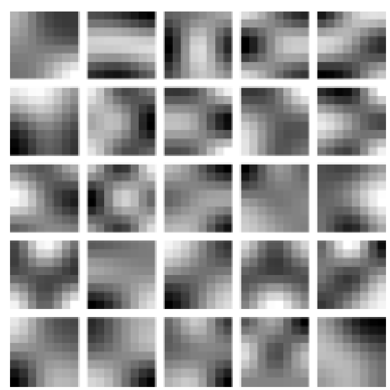
RESULTS

Validation - ICA

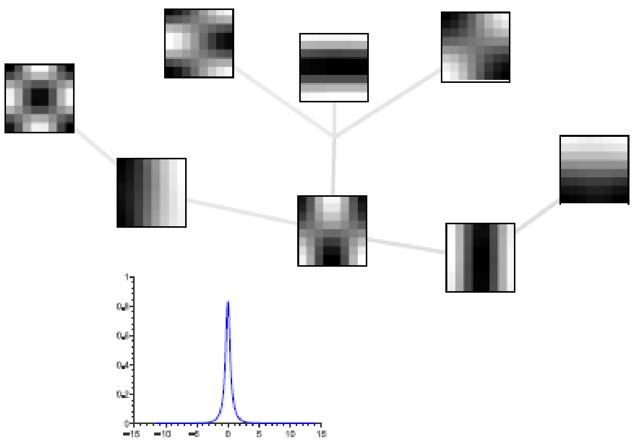
Generative model



Samples

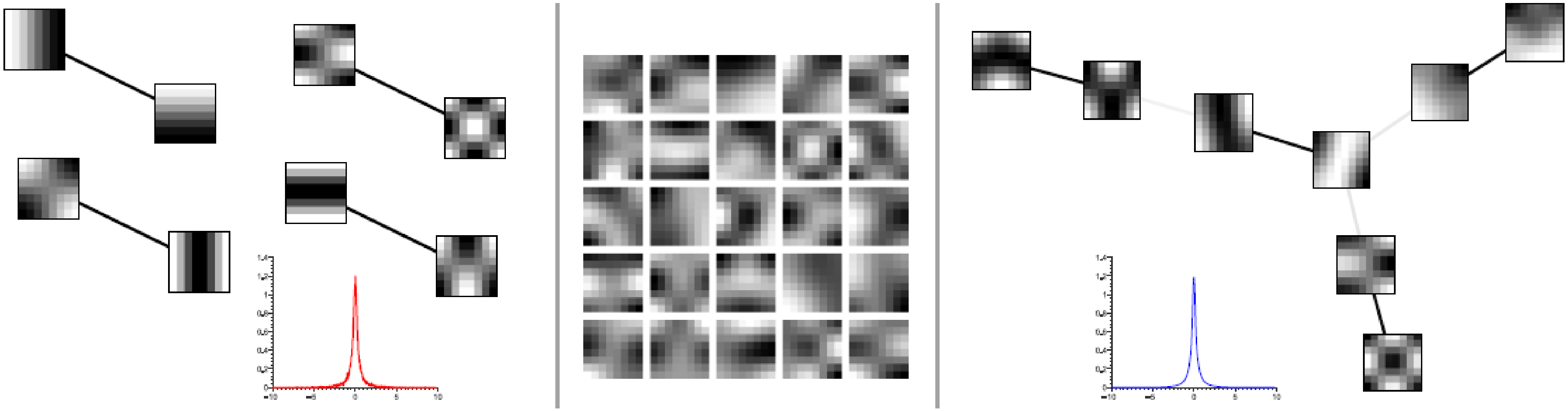


Learned tree model



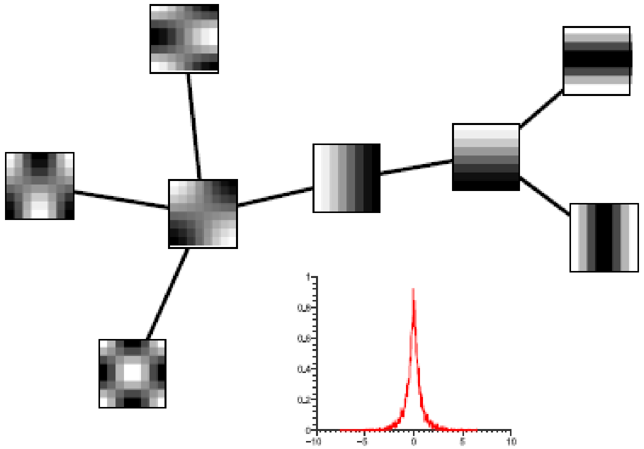
VALIDATION - ISA

Generative model | Samples | Learned tree model

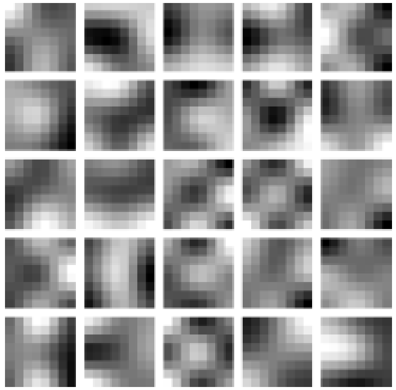


VALIDATION – TREE MODEL

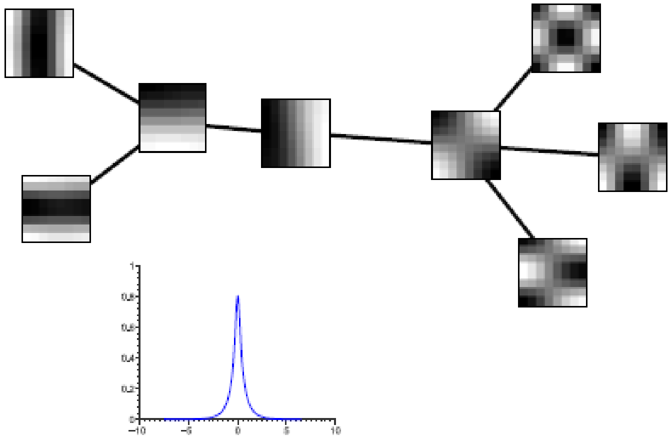
Generative model



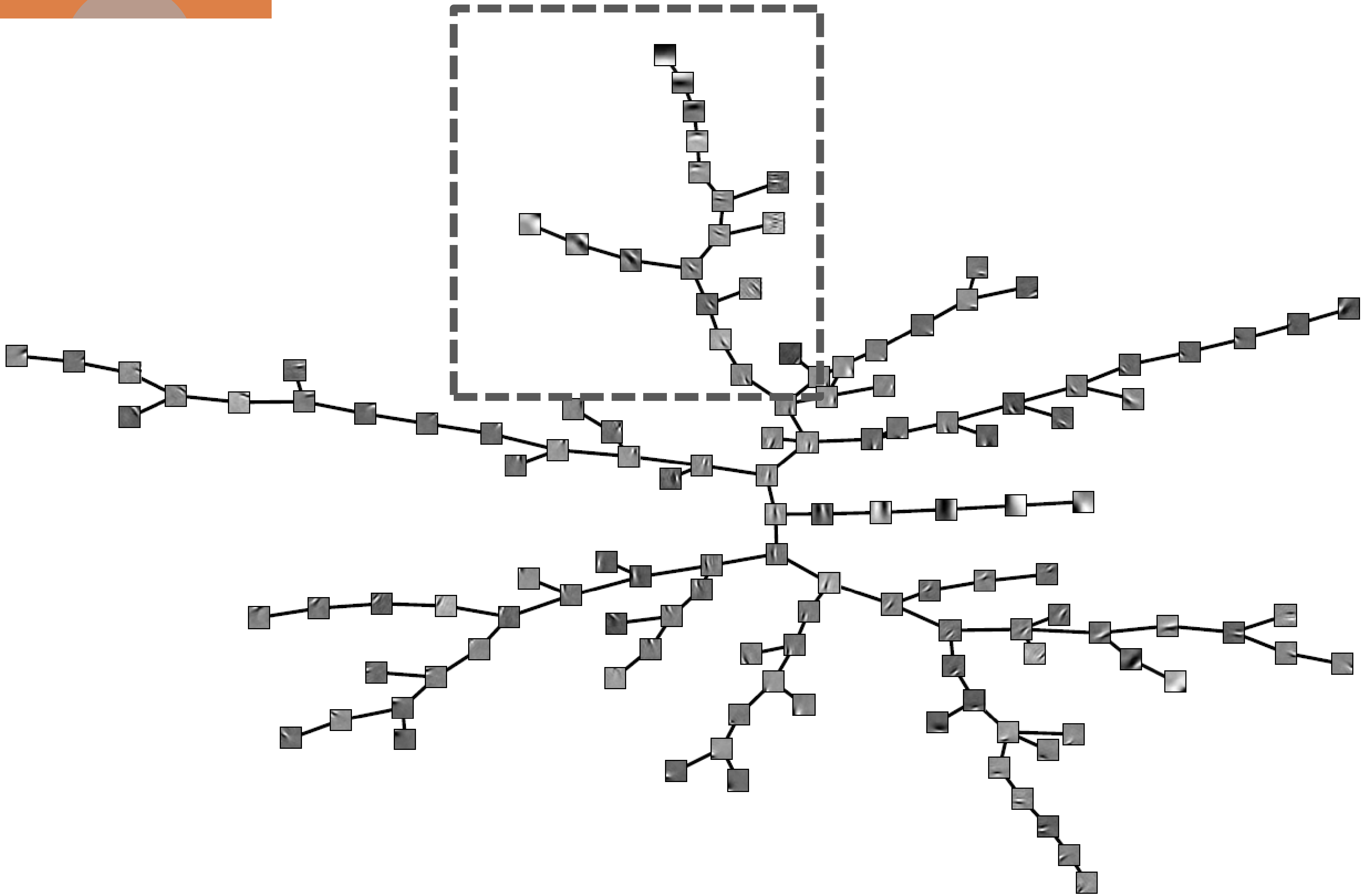
Samples

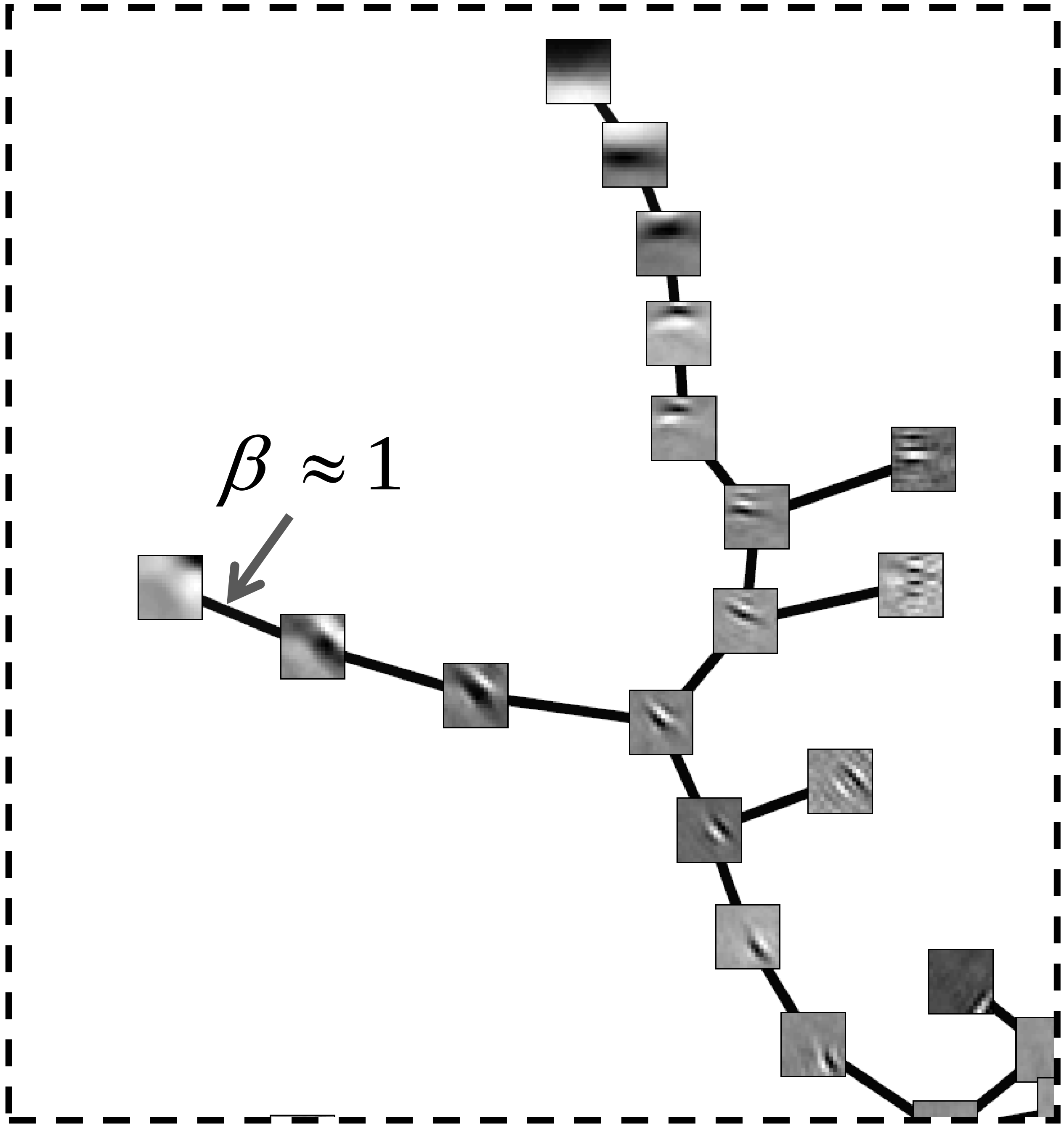


Learned tree model

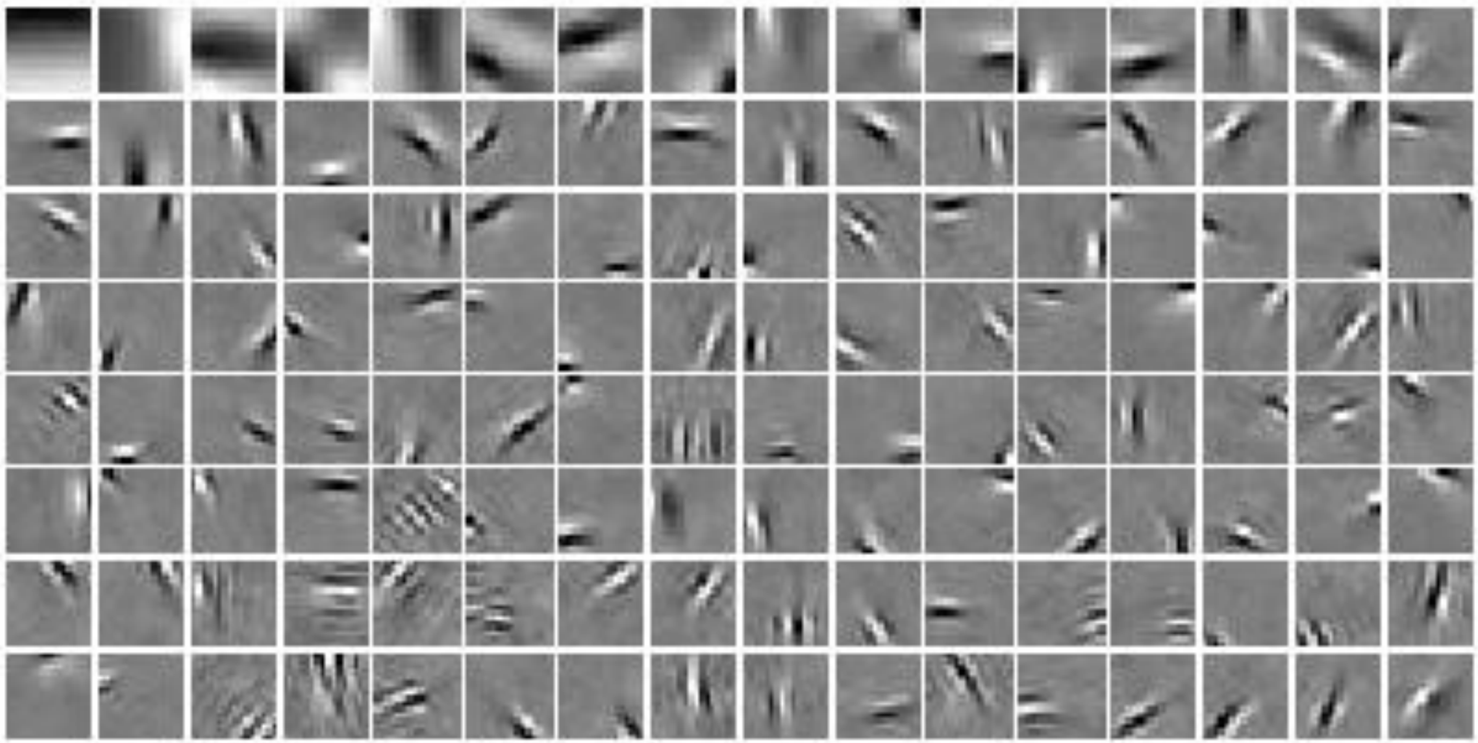


Learned Tree Structure



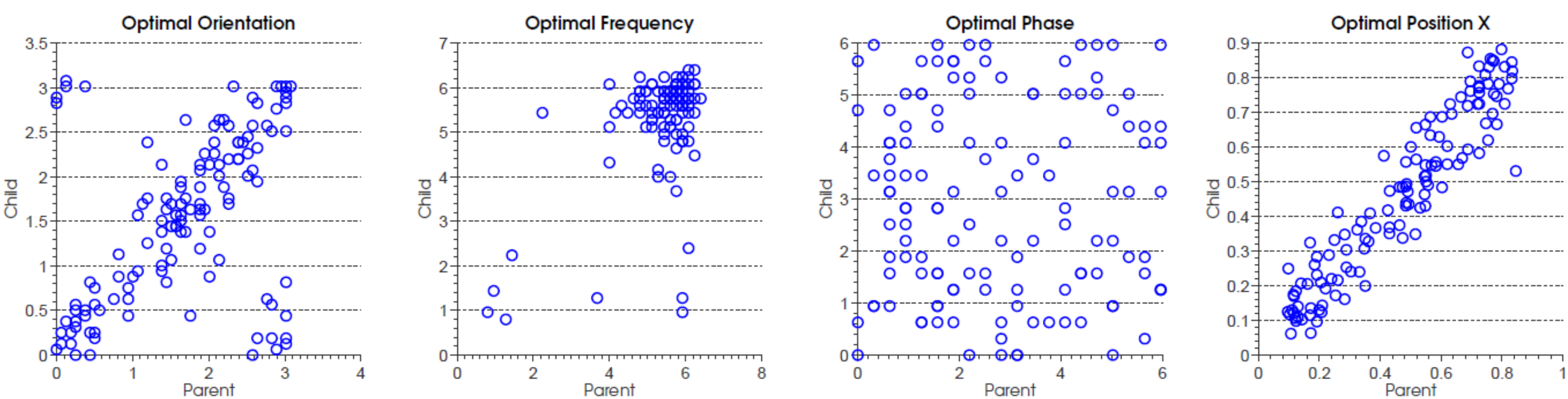

$\beta \approx 1$

Learned Edge Features



Correlations Between Pairs

- Orientation, Frequency and Position – High Correlation
- Phase is uncorrelated
- Akin to complex cell models

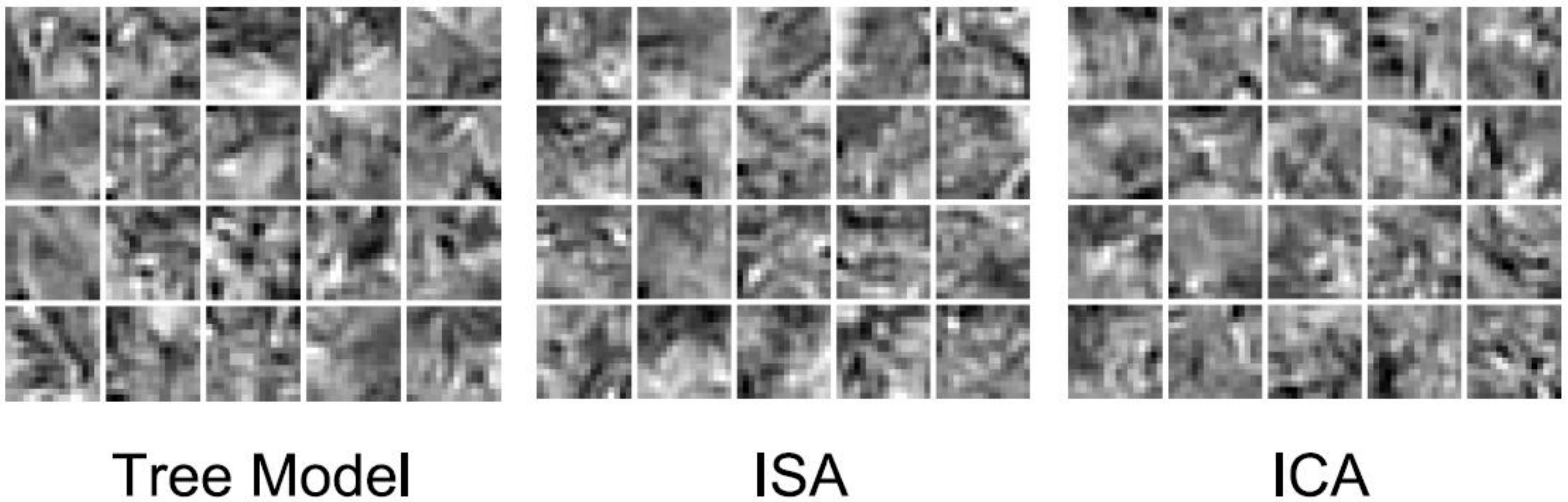


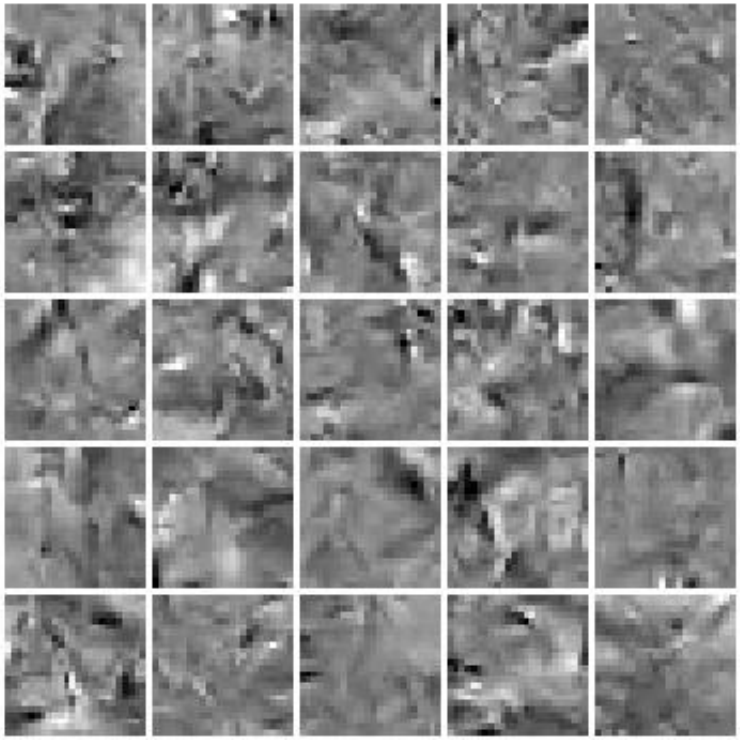
Comparison to other models - Likelihood

- Likelihood comparison – over an unseen test set

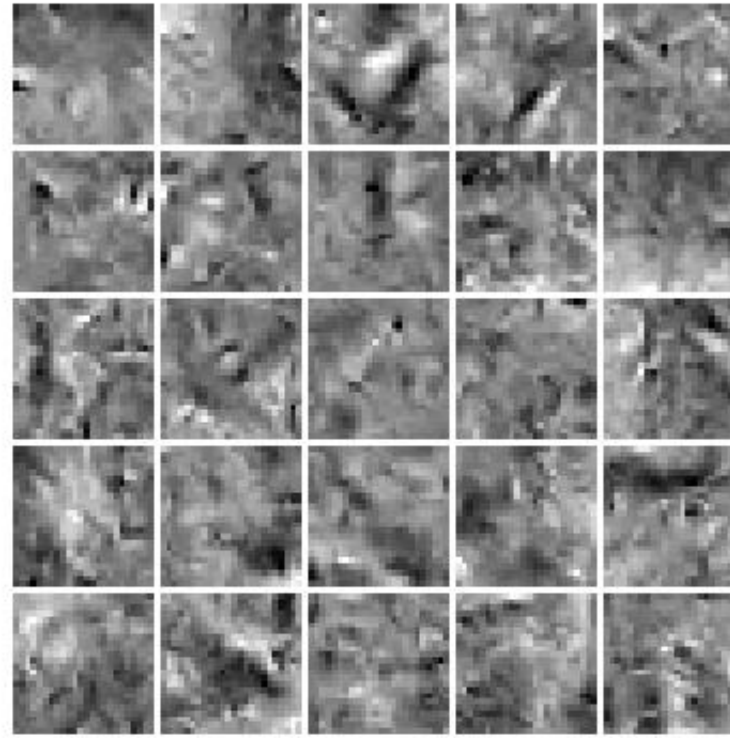
Model	Log Likelihood
Marginal PCA	-162.5
Marginal ICA	-157
ISA	-159.4
Our model	**-144.8**

Comparison (cont.) – Samples

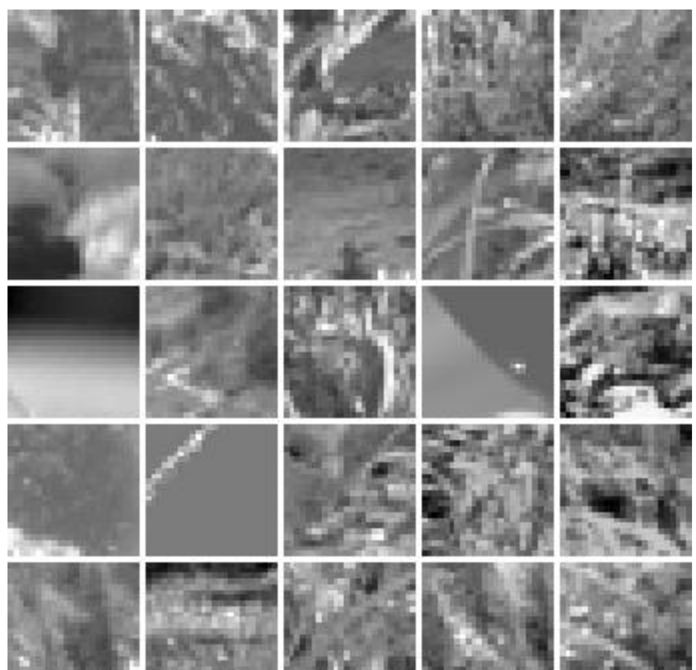




Tree



ICA



Natural Images

Conclusions

- Learned components are edge filters, even though we assumed dependence
- Learned conditional density is bowtie
- Learned connections between filters give "complex cells" – orientation tuned and phase invariant

THANKS!