Probabilistic Symbolic Model Checking: Theory and Practice

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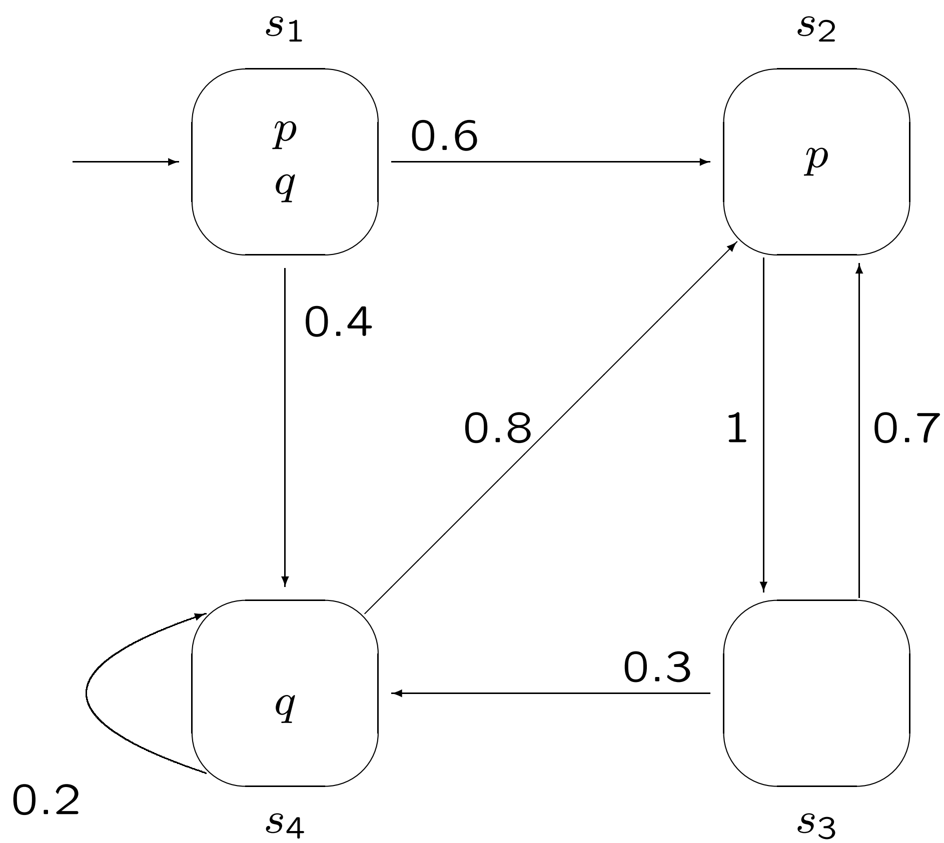
Overview

	Discret Time	Continuous Time
Deterministic	Discrete Markov Chain PCTL	Continuous Markov Chain CSL
Nondeterminstic	Markove Decision Process pCTL	Probabilistic Timed Automata

Discrete Markov Chain



PCTL

$s \models_D a$	iff $a \in L(s)$
$s \models_D \neg f$	iff not $s \models_D f$
$s \models_D f_1 \wedge f_2$	iff $s \models_D f_1$ and $s \models_D f_2$
$\sigma \models_D f_1 U^{\leq t} f_2$	iff there exists an $i \leq t$ such that $\sigma[i] \models_D f_2$ and $\sigma[j] \models_D f_1$, for all $j : 0 \leq j < i$.
$s \models_D f_1 U^{\leq t}_{\geq p} f_2$	iff $\mu_s^D(\{\sigma \mid \sigma[0] = s \wedge \sigma \models_D f_1 U^{\leq t} f_2\}) \geq p$.
$s \models_D f_1 U^{\leq t}_{> p} f_2$	iff $\mu_s^D(\{\sigma \mid \sigma[0] = s \wedge \sigma \models_D f_1 U^{\leq t} f_2\}) > p$.

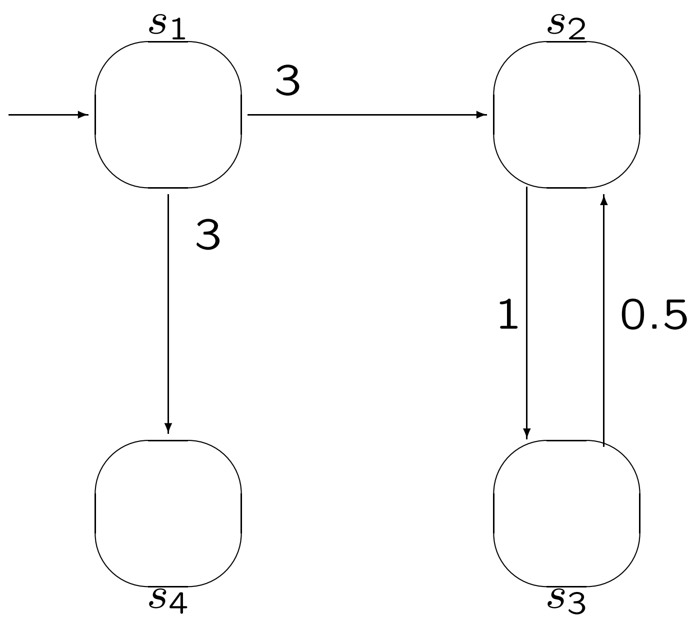
$$
\begin{array}{lcl}
f_1 \vee f_2 & \equiv & \neg(\neg f_1 \wedge \neg f_2) \\
f_1 \rightarrow f_2 & \equiv & \neg f_1 \vee f_2 \\
f_1 u^{\leq t}_{\geq p} f_2 & \equiv & \neg(\neg f_2 U^{\leq t}_{> 1-p} \neg(f_1 \vee f_2)) \\
f_1 u^{\leq t}_{> p} f_2 & \equiv & \neg(\neg f_2 U^{\leq t}_{\geq 1-p} \neg(f_1 \vee f_2)) \\
G^{\leq t}_{\geq p} f & \equiv & f \ u^{\leq t}_{\geq p} false \\
F^{\leq t}_{\geq p} f & \equiv & true \ U^{\leq t}_{\geq p} f
\end{array}
$$

Markov Decision Process and pCTL

$$
\begin{array}{lll}
P & \subseteq & Stat \\
\phi, \psi \in Stat & \Longrightarrow & \phi \wedge \psi, \neg\phi \in Stat \\
\phi \in Seq & \Longrightarrow & A\phi, E\phi, \mathbf{P}_{\bowtie a}\phi \in Stat \\
\phi \in Stat & \Longrightarrow & \phi \in Seq \\
\phi, \psi \in Seq & \Longrightarrow & \phi \wedge \psi, \neg\phi, \Box\, \phi, \Diamond\phi, \phi U \psi \in Seq
\end{array}
$$

$$
\begin{array}{lll}
s \models A\phi & \text{iff} & \forall\omega \in \Omega_s \cdot \omega \models \phi \\
s \models E\phi & \text{iff} & \exists\omega \in \Omega_s \cdot \omega \models \phi \\
s \models \mathbf{P}_{\geq a}\phi & \text{iff} & \mu_s^-(\{\omega \in \Omega_s \mid \omega \models \phi\}) \geq a \\
s \models \mathbf{P}_{\leq a}\phi & \text{iff} & \mu_s^+(\{\omega \in \Omega_s \mid \omega \models \phi\}) \leq a
\end{array}
$$

Continuous Markov Chain



$$\mathbf{Q} = \begin{pmatrix} 0 & 3 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

Probability to make a transition within t time units

$p(s, s') = 1 - e^{-\mathbf{Q}(s,s')\cdot t} \quad t > 0$

$\mathbf{E}(s) = \sum_{s' \in S} \mathbf{Q}(s, s')$

$\mathbf{P}(s, s') = \mathbf{Q}(s, s') / \mathbf{E}(s)$

What we didn't do

- MTBDD and sparse matrix in probabilistic model checking
- pCTL*! PCTL*? CSL*?
- Other systems
- Automata-theoretic approach to probabilistic model checking
- Probabilistic Timed Automata